2 LIFE MATHEMATICS

Learning Objectives

- To recall direct and inverse proportions.
- To know compound variation and do problems on it.
- To solve time and work problems.

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2.1 Introduction

The following conversation takes place in the Math class of Std VIII.

Teacher: Students, before we could learn about what Compound Variation is, let me ask you a few questions on direct and inverse (indirect) proportions which you have already learnt in Std VII. Can anyone of you tell me what direct proportion is?

Bharathi: Yes, teacher. If one quantity increases or decreases depending on the increase or decrease of another quantity simultaneously, then it is direct proportion.

Teacher: Good Bharathi, give me an example too.

Bharathi: Well teacher, if I plan to give 2 pens to each of my friends in the birthday party, the number of pens to be bought will be in direct proportion with the number of friends who will attend the party. The following table will help us understand clearly, teacher.

Number of friends	1	2	5	12	15
Number of pens	2	4	10	24	30

Teacher: Very good example Bharathi. Students give her a big hand. (The class applauds). Mukesh, can you tell about inverse proportion?

Mukesh: Yes teacher, if our class of 30 students goes on streets in our village for health awareness campaign in an orderly manner then, we can see an inverse proportion in the number of rows and columns, Teacher, this is easily understood from the following table.

Number of students in columns	1	2	3	5	6
Number of students in rows	30	15	10	6	5

Teacher: Fine Mukesh, you have explained it nicely with a good example.

Mukesh: We can map a few of these arrangements as  and  also see the opposite variations in rows and columns, teacher.

Teacher: Well done Mukesh. Students, I hope you have now understood clearly by these two examples about direct and inverse proportions which you have already learnt in Std VII. Let me now explain what a Compound Variation is? Some problems may involve a chain of two or more variations in them what we call as **Compound Variation.**

Ragini: Teacher, Can you explain the Compound Variation with an example?

Teacher: Yes, Ragini, I will. Before I could explain that, let me ask you all another question. If Kani can finish a given work in 2 hours and Viji in 3 hours, then in what time can they finish it working together?

Bharathi: I think, they will finish it in $2\frac{1}{2}$ hours. Am I correct teacher?

Teacher: Not really Bharathi. I will tell you the correct answer. These types of questions which come under the heading **Time and Work** need some explanation and we will learn all these topics in this term.

Now, let us recall the concepts about the direct and inverse proportions.

2.2 Direct Proportion

If two quantities are such that an increase or decrease in one quantity makes a corresponding increase or decrease (same effect) in the other quantity, then they are said to be in direct proportion or said to vary directly. In other words, x and y are said to vary directly if $\frac{x}{y} = k$ always, where k is a positive constant.

Examples of Direct Proportion:

1. Distance –Time (under constant speed): If distance increases, the time taken to reach that distance will also increase and vice- versa.
2. Purchase – Spending: If the purchase on utilities for a family during the festival time increases,the spending limit also increases and vice versa.
3. Work Time – Earnings: If the number of hours worked is less, the pay earned will be less and vice-versa.

2.3 Inverse Proportion

If two quantities are such that an increase or decrease in one quantity makes a corresponding decrease or increase (opposite effect) in the other quantity, then they are said to be in inverse (indirect) proportion or said to vary inversely. In other words, x and y are said to vary inversely, if $xy = k$ always, where k is a positive constant.

Examples of Inverse Proportion:

1. Price – Consumption: If the price of an article increases, then its consumption will naturally decrease and vice-versa.

2. Workers – Time: If more workers are employed to complete a work, then the time taken to complete will be less and vice-versa.
3. Speed – Time (Fixed Distance): If we travel with less speed,the time taken to cover a given distance will be more and vice-versa.

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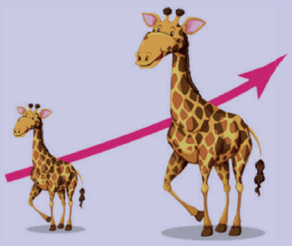 	
The growth of a giraffe over time is an example for direct proportion and see-saw is an example for inverse proportion	If 3 persons A, B and C can do a work in x, y and z days respectively, then the ratio in which their wages will be distributed to them is $\frac{1}{x} : \frac{1}{y} : \frac{1}{z}$.

Try these

Classify the given examples as direct or inverse proportion:

(i) Weight of pulses to their cost.
(ii) Distance travelled by bus to the price of ticket.
(iii) Speed of the athelete to cover a certain distance.
(iv) Number of workers employed to complete a construction in a specified time.
(v) Volume of water flown through a pipe to its pressure.
(vi) Area of a circle to its radius.

Use the concept of direct and inverse proportions and try to answer the following questions:

1. A student can type 21 pages in 15 minutes. At the same rate, how long will it take the student to type 84 pages?
2. The weight of an iron pipe varies directly with its length. If 8 feet of an iron pipe weighs 3.2 kg, find the proportionality constant k and determine the weight of a 36 feet iron pipe.
3. A car covers a distance of 765 km in 51 litres of petrol. How much distance would it cover in 30 litres of petrol?
4. If x and y vary inversely and $x = 24$ when $y = 8$, find x when $y = 12$.
5. If 35 women can do a piece of work in 16 days, in how many days will 28 women do the same work?

6. A farmer has food for 14 cows which can last for 39 days. How long would the food last, if 7 more cows join his cattle?

7. Identify the type of proportion and fill in the blank boxes:

x	1	2		4	6	8		12	15		24
y	20		60		120		180		300	360	

8. Identify the type of proportion and fill in the blank boxes:

x	1	2		4	6	8		12		18	24
y	144		48		24		16		9	8	

2.4 Compound Variation

There will be problems which may involve a chain of two or more variations in them. This is called as compound variation. The different possibilities of two variations are: **Direct-Direct, Direct-Inverse, Inverse- Direct, Inverse- Inverse.**

Note

There are situations where neither direct proportion nor indirect proportion can be applied. For example, if one can see a parrot at a distance through one eye, it does not mean that he can see two parrots at the same distance through both the eyes. Also, if it takes 5 minutes to fry a vadai, it does not mean that 20 vadais will take 100 minutes to fry!

Let us now solve a few problems on compound variation. Here, we compare the known quantity with the unknown (x). There are a few methods in practice by which problems on compound variation are solved. They are:

2.4.1 Proportion Method:

In this method, we shall compare the given data and find whether they are in direct or indirect proportion. By finding the proportion, we can use the fact that

the product of the extremes = the product of the means

and get the value of the unknown (x).

2.4.2 Multiplicative Factor Method:

Illustration:

Men	Hours	Days
D a / x I	D c / d	e / f I

Here, the unknown (x) in men is compared to the known, namely hours and days. If men and hours are in direct proportion (*D*) then, take the multiplying factor as is $\frac{d}{c}$ (take the reciprocal). Also, if men and days are in inverse proportion (*I*), then take the multiplying factor as $\frac{e}{f}$ (no change). Thus, we can find the unknown (x) in men as $x = a \times \frac{d}{c} \times \frac{e}{f}$.

Note

- When the number of days is constant, work and persons are directly proportional to each other and vice-versa.

  i.e., increase (↑) in work means increase (↑) in persons with same number of days.
- When the number of persons is constant, work and days are directly proportional to each other and vice-versa.

  i.e., increase (↑) in work means increase (↑) in days with same number of persons.
- When the work is constant, the number of persons and days are inversely proportional to each other and vice-versa.

  i.e., increase (↑) persons means decrease (↓) in days with constant work.

2.4.3 Formula Method:

Identify the data from the given statement as Persons (P), Days (D), Hours (H) and Work (W) and use the formula,

$$\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$$

where the suffix 1 contains the complete data from the first statement of the given problem and the suffix 2 contains the unknown data in the second statement to be found out in the problem. That is, this formula says, P_1 men doing W_1 units of work in D_1 days working H_1 hours per day is the same as P_2 men doing W_2 units of work in D_2 days working H_2 hours per day. Identifying the work W_1 and W_2 correctly is more important in these problems. This method will be easy for finding the unknown (x) quickly.

Example 2.1 (Direct – Direct Variation)

If a company pays ₹6 lakh for 15 workers for 20 days, what would it pay for 5 workers for 12 days?

Solution:

Proportion Method:

Workers	Payment (Work)	Days
15	6	20
D	*D* *D*	*D*
5	x	12

Here, the unknown is the payment (x). It is to be compared with the workers and the days.

Step 1:

Here, less days means less payment. So, it is a direct proportion.

$\therefore$ The proportion is $20:12::6:x$ $\longrightarrow$ (1)

Step 2:

Also, less workers means less payment. So, it is a direct proportion again.

$\therefore$ The proportion is $15:5::6:x$ $\longrightarrow$ (2)

Step 3:

Combining (1) and (2)

$$\left.\begin{matrix} 20:12 \\ 15:5 \end{matrix}\right\} :: 6:x$$

We know that the product of the extremes = the product of the means

Extremes		Means		Extremes
20	:	12 : 6	:	x
15	:	5		

So, $20\times15\times x=12\times6\times5 \Rightarrow x=\dfrac{12\times6\times5}{20\times15}$ = ₹ 1.2 lakh.

Multiplicative Factor Method:

Workers	Payment (Work)	Days
15	6	20
D 5	D x D	D 12

Here, the unknown is the payment (x). It is to be compared with the workers and the days.

Step 1:

Here, less days means less payment. So, it is a direct proportion.

$\therefore$ The multiplying factor is $\dfrac{12}{20}$ (take the reciprocal).

Step 2:

Also, less workers means less payment. So, it is a direct proportion again.

$\therefore$ The multiplying factor is $\dfrac{5}{15}$ (take the reciprocal).

Step 3:

$$\therefore\ x=6\times\frac{12}{20}\times\frac{5}{15}$$

$$x = ₹\ 1.2 \text{ lakh}$$

Formula Method

Here, $P_1=15, D_1=20$ and $W_1=6$

$P_2=5, D_2=12$ and $W_2=x$

Using the formula, $\dfrac{P_1\times D_1}{W_1}=\dfrac{P_2\times D_2}{W_2}$

We have, $\dfrac{15\times20}{6}=\dfrac{5\times12}{x}$

$$\Rightarrow x=\frac{5\times12\times6}{15\times20}=₹\ 1.2\ \text{lakh}.$$

Example 2.2 (Direct – Inverse Variation)

A mat of length 180 m is made by 15 women in 12 days. How long will it take for 32 women to make a mat of length 512 m?

Solution:

Proportion Method:

Length (Work)	Women	Days
180	15	12
D	*I*	*D* *I*
512	32	x

Here, the unknown is the days (x). It is to be compared with the length and the women.

Step 1:

Here, more length means more days. So, it is a direct proportion.

$\therefore$ The proportion is 180 : 512 :: 12 : x $\longrightarrow$ (1)

Step 2:

Also, more women means less days. So, it is an inverse proportion.

$\therefore$ The proportion is 32 : 15 :: 12 : x $\longrightarrow$ (2)

Step 3:

Combining (1) and (2)

$$\left.\begin{matrix}180:512\\32:15\end{matrix}\right\}::12:x$$

We know that the product of the extremes = the product of the means

Extremes		Means		Extremes
180	:	512 : 12	:	x
32	:	15		

So, $180\times32\times x=512\times12\times15 \Rightarrow x=\dfrac{512\times12\times15}{180\times32}=16$ days.

Multiplicative Factor Method:

Length (Work)	Women	Days
180	15	12
D	*I*	*D* *I*
512	32	x

Here, the unknown is the days (x). It is to be compared with the length and the women.

Unit -2 Life Mathematics.indd 37 07-11-2019 16:21:52

Step 1:

Here, more length means more days. So, it is a direct proportion.

$\therefore$ The multiplying factor is $\frac{512}{180}$ (take the reciprocal).

Step 2:

Also, more women means less days. So, it is an inverse proportion.

$\therefore$ The multiplying factor is $\frac{15}{32}$ (no change).

Step 3:

$\therefore x = 12 \times \frac{512}{180} \times \frac{15}{32} = 16$ days.

Formula Method:

Here, $P_1 = 15,\ D_1 = 12$ and $W_1 = 180$

$P_2 = 32,\ D_2 = x$ and $W_2 = 512$

Using the formula, $\frac{P_1 \times D_1}{W_1} = \frac{P_2 \times D_2}{W_2}$

We have, $\frac{15 \times 12}{180} = \frac{32 \times x}{512}$

$$\Rightarrow\ 1 = \frac{32 \times x}{512} \quad \Rightarrow\ x = \frac{512}{32} = 16 \text{ days.}$$

Remark: Students may answer in any of the three given methods dealt here.

Try these

1. When $x = 5$ and $y = 5$ find k, if x and y vary directly.
2. When x and y vary inversely, find the constant of variation when $x = 64$ and $y = 0.75$
3. You draw a circle of a given radius. Then, draw its radii in such a way that the angles between any pair of radii are equal. You start with drawing 3 radii and end with drawing 12 radii in the circle. Prepare a table for the number of radii to the angle between a pair of consecutive radii and check whether they are in inverse proportion. What is the proportionality constant?

Think

(i) When x and y are in direct proportion and if y is doubled, then what happens to x?

(ii) If $\frac{x}{y-x} = \frac{6}{7}$ what is $\frac{x}{y}$?

Example 2.3 (Inverse – Direct Variation)

If 81 students can do a painting on a wall of length 448 m in 56 days. How many students can do the painting on a similar type of wall of length 160 m in 27 days?

Solution:

Multiplicative Factor Method:

Students	Days	Length of the wall (Work)
81	56	448
I *D*	*I*	*D*
x	27	160

Step 1:

Here, less days means more students. So, it is an inverse variation.

$\therefore$ The multiplying factor is $\frac{56}{27}$.

Step 2:

Also, less length means less students. So, it is a direct variation.

$\therefore$ The multiplying factor is $\frac{160}{448}$.

Step 3:

$$\therefore x = 81 \times \frac{56}{27} \times \frac{160}{448}$$

$x = 60$ students.

Formula Method:

Here, $P_1 = 81$, $D_1 = 56$ and $W_1 = 448$

$P_2 = x$, $D_2 = 27$ and $W_2 = 160$

Using the formula, $\frac{P_1 \times D_1}{W_1} = \frac{P_2 \times D_2}{W_2}$

We have, $\frac{81 \times 56}{448} = \frac{x \times 27}{160}$

$$\Rightarrow x = \frac{81 \times 56}{448} \times \frac{160}{27}$$

$x = 60$ students.

Example 2.4 (Inverse – Inverse Variation)

If 48 men working 7 hours a day can do a work in 24 days, then in how many days will 28 men working 8 hours a day can complete the same work?

Solution:

Multiplicative Factor Method:

Men	Hours	Days
I 48	7 *I*	*I* 24 *I*
28	8	x

Step 1:

Here, less men means more days. So, it is an inverse variation.

$\therefore$ The multiplying factor is $\frac{48}{28}$.

Step 2:

Also, more hours means less days. So, it is an inverse variation.

$\therefore$ The multiplying factor is $\frac{7}{8}$.

Step 3:

$$\therefore x = 24 \times \frac{48}{28} \times \frac{7}{8} = 36 \text{ days.}$$

Formula Method:

Here, $P_1 = 48, D_1 = 24, H_1 = 7$ and $W_1 = 1$ (Why?)

$P_2 = 28, D_2 = x, H_2 = 8$ and $W_2 = 1$ (Why?)

Using the formula, $\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$

We have, $\frac{48 \times 24 \times 7}{1} = \frac{28 \times x \times 8}{1}$

$$\Rightarrow x = \frac{48 \times 24 \times 7}{28 \times 8} = 36 \text{ days.}$$

Try these

Identify the different variations present in the following questions:

1. 24 men can make 48 articles in 12 days. Then, 6 men can make ______ articles in 6 days.
2. 15 workers can lay a road of length 4 km in 4 hours. Then, ______ workers can lay a road of length 8 km in 8 hours.
3. 25 women working 12 hours a day can complete a work in 36 days. Then, 20 women must work ______ hours a day to complete the same work in 30 days.
4. In a camp, there are 420 kg of rice sufficient for 98 persons for 45 days. The number of days that 60 kg of rice will last for 42 persons is ______.

Example 2.5

If 15 men take 40 days to complete a work, how long will it take if 15 more men join them to complete the same work?

Solution:

If 15 men can complete the work in 40 days, then the work measured in terms of person days = $15 \times 40 = 600$ person days.

If the same work is to be done by 30 (15 + 15) men, then the number of days they will take is $\frac{600}{30} = 20$ days.

Note

- The concept of *person days* is important here. The number of *persons* multiplied by the number of *days* required to complete the work gives the *person days.* Here, work is measured in terms of *person days.*
- If x women or y men can complete a piece of work in p days, then a women and b men can complete the same work in $\frac{xyp}{xb+ya}$ (or) $\frac{p}{\frac{a}{x}+\frac{b}{y}}$ days.

Example 2.6

6 women or 8 men can construct a room in 86 days. How long will it take for 7 women and 5 men to do the same type of room?

Solution:

Person days Method:

Here, let M and W denote a men and a women respectively.

Given that, 6W = 8M $\Rightarrow 1\text{W} = \frac{8}{6}\text{M} = \frac{4}{3}\text{M}$.

Now, $7\text{W}+5\text{M} = 7\times\frac{4}{3}\text{M}+5\text{M} = \frac{43\text{M}}{3}$

If 8M can construct the room in 86 days, then $\frac{43\text{M}}{3}$ can construct the same type of room in $8\text{M}\times 86 \div \frac{43\text{M}}{3} = 8\text{M}\times 86\times\frac{3}{43\text{M}} = 48$ days.

Formula Method:

Required time to construct the room = $\frac{xyp}{xb+ya}$

$$= \frac{6\times8\times86}{6\times5+8\times7} = \frac{6\times8\times86}{30+56} = \frac{6\times8\times86}{86} = 48 \text{ days.}$$

(or)

Required time to construct the room = $\frac{p}{\frac{a}{x}+\frac{b}{y}}$

$$= \frac{86}{\frac{7}{6}+\frac{5}{8}} = \frac{86\times48}{86} = 48 \text{ days.}$$

2.5 Time and Work

Work to be done is usually considered as one unit. Work can be in any form like building a wall, making a road, filling or emptying a tank, or even eating a certain amount of food.

Time is measured in hours, days etc., Certain assumptions are made that the work so done is uniform and each person shares the same work time in case of group work in completing the work.

Unitary Method:

If two persons X and Y can do some work individually in a and b days, then their one day's work is $\frac{1}{a}$ and $\frac{1}{b}$ respectively.

Also, their one day's work together = $\frac{1}{a}+\frac{1}{b} = \frac{a+b}{ab}$

Thus, X and Y together can complete the work in $\frac{ab}{a+b}$ days.

Example 2.7

A and B together can do a piece of work in 16 days and A alone can do it in 48 days. How long will B done take to complete the work?

Solution:

$(A+B)$'s 1 day's work $= \frac{1}{16}$

A's 1 day's work $= \frac{1}{48}$

$\therefore$ B's 1 day's work $= \frac{1}{16} - \frac{1}{48}$

$= \frac{3-1}{48} = \frac{2}{48} = \frac{1}{24}$

$\therefore$ B alone can complete the work in 24 days.

Note

The time taken to complete a work or task depends on various factors such as number of persons, their capacity to do the work, the amount of work and the time spent per day for the completion of work.

DO YOU KNOW?

If A is $\frac{a}{b}$ times as good a worker as B, then A will take $\frac{b}{a}$ of the time taken by B to complete the work.

Example 2.8

A works 3 times as fast as B and is able to complete a task in 24 days less than the days taken by B. Find the time in which they can complete the work together.

Solution:

If B does the work in 3 days, then A will do it in 1 day. That is, the difference is 2 days. Here, given that the difference between A and B in completing the work is 24 days. Therefore, A will take $\frac{24}{2} = 12$ days and B will take $3 \times 12 = 36$ days to complete the work separately.

Hence, the time taken by A and B together to complete the work = $\frac{ab}{a+b}$ days.

$$= \frac{12 \times 36}{12+36} = \frac{12 \times 36}{48} = 9 \text{ days.}$$

Example 2.9

P and Q can do a piece of work in 20 days and 30 days respectively. They started the work together and Q left after some days of work and P finished the remaining work in 5 days. After how many days from the start did Q leave?

Solution:

$$\text{P's 1 day's work} = \frac{1}{20} \text{ and Q's 1 day's work} = \frac{1}{30}$$

$$\text{P's work for 5 days} \quad = \frac{1}{20} \times 5 = \frac{5}{20} = \frac{1}{4}$$

Therefore, the remaining work $= 1 - \frac{1}{4} = \frac{3}{4}$ (Total work is always 1)

This remaining work was done by both P and Q.

Work done by P and Q in a day $= \frac{1}{20} + \frac{1}{30} = \frac{5}{60} = \frac{1}{12}$

Therefore, the number of days they worked together $= \dfrac{3/4}{1/12} = \frac{3}{4} \times \frac{12}{1} = 9$ days

So, Q left after 9 day from the days the work started.

Example 2.10

A and B can do a piece of work in 12 days and 9 days respectively. They work on alternate days starting with A on the first day. In how many days will the work be completed?

Solution:

Since they work on alternate days, let us consider a period of two days.

In the period of 2 days, work done by A and B = $\frac{1}{12} + \frac{1}{9} = \frac{7}{36}$

If we consider 5 such time periods for the fraction $\frac{7}{36}$ (we consider 5 periods because 7 goes 5 times completely in 36),

work done by A and B in $5 \times 2\ (=10)$ days = $5 \times \frac{7}{36} = \frac{35}{36}$

Therefore, the remaining work $= 1 - \frac{35}{36} = \frac{1}{36}$.

This is done by A (why?) in $\frac{1}{36} \times 12 = \frac{1}{3}$ days

So, the total time taken = 10 days + $\frac{1}{3}$ days $= 10\frac{1}{3}$ days.

2.6 Sharing of the money for work

When a group of people do some work together, based on their individual work, they get a share of money themselves. In general, **money earned** is shared by people, who worked together, in the ratio of the **total work** done by each of them.

DO YOU KNOW?

- If the ratio of the time taken by A and B in doing a work is $x : y$, then the ratio of work done by A and B is $\frac{1}{x} : \frac{1}{y} = y : x$. This is the ratio for their separate wages too.
- If three persons A, B and C can do a work in x, y and z days respectively, then the ratio in which their wages will be distributed to them is $\frac{1}{x} : \frac{1}{y} : \frac{1}{z}$.

Example 2.11

X, Y and Z can do a piece of job in 4, 6 and 10 days respectively. If X,Y and Z work together to complete, then find their separate shares if they will be paid ₹ 3100 for completing the job.

Solution:

Since they all work for the same number of days, the ratio in which they share the money is equal to the ratio of their work done per day.

That is, $\frac{1}{4}:\frac{1}{6}:\frac{1}{10}=\frac{15}{60}:\frac{10}{60}:\frac{6}{60}=15:10:6$

Here, the total parts = 15 + 10 + 6 = 31

Hence, A's share = $\frac{15}{31}\times 3100=₹1500$, B's share = $\frac{10}{31}\times 3100=₹1000$ and

C's share is ₹ 3100 – (₹ 1500 + ₹ 1000)= ₹ 600.

Try these

1. Vikram can do one-third of work in p days. He can do $\frac{3}{4}$ th of work in ________ days.
2. If m persons can complete a work in n days ,then $4m$ persons can complete the same work in ________days and $\frac{m}{4}$ persons can complete the same work in ________ days.

Exercise 2.1

1. Fill in the blanks:

(i) A can finish a job in 3 days where as B finishes it in 6 days. The time taken to complete the job together is ____________days.

(ii) If 5 persons can do 5 jobs in 5 days, then 50 persons can do 50 jobs in _________ days.

(iii) A can do a work in 24 days. A and B together can finish the work in 6 days. Then B alone can finish the work in _________ days.

(iv) A alone can do a piece of work in 35 days. If B is 40% more efficient than A, then B will finish the work in ____________days.

(v) A alone can do a work in 10 days and B alone in 15 days. They undertook the work for ₹200000. The amount that A will get is _________.

2. 210 men working 12 hours a day can finish a job in 18 days. How many men are required to finish the job in 20 days working 14 hours a day?
3. A cement factory makes 7000 cement bags in 12 days with the help of 36 machines. How many bags can be made in 18 days using 24 machines?
4. A soap factory produces 9600 soaps in 6 days working 15 hours a day. In how many days will it produce 14400 soaps working 3 hours more a day?
5. If 6 container lorries can transport 135 tonnes of goods in 5 days, how many more lorries are required to transport 180 tonnes of goods in 4 days?
6. A can do a piece of work in 12 hours, B and C can do it 3 hours whereas A and C can do it in 6 hours. How long will B alone take to do the same work?
7. A and B can do a piece of work in 12 days, while B and C can do it in 15 days whereas A and C can do it in 20 days. How long would each take to do the same work?
8. Carpenter A takes 15 minutes to fit the parts of a chair while Carpenter B takes 3 minutes more than A to do the same work. Working together, how long will it take for them to fit the parts for 22 chairs?
9. A man takes 10 days to finish a job where as a woman takes 6 days to finish the same job. Together they worked for 3 days and then the woman left. In how many days will the man complete the remaining job?
10. A is thrice as fast as B. If B can do a piece of work in 24 days, then find the number of days they will take to complete the work together.

Exercise 2.2

Miscellaneous and Practice Problems

1. 5 boys or 3 girls can do a science project in 40 days. How long will it take for 15 boys and 6 girls to do the same project?
2. If 32 men working 12 hours a day can do a work in 15 days, how many men working 10 hours a day can do double that work in 24 days?
3. Amutha can weave a saree in 18 days. Anjali is twice as good a weaver as Amutha. If both of them weave together, in how many days can they complete weaving the saree?
4. A, B and C can complete a work in 5 days. If A and C can complete the same work in $7\frac{1}{2}$ days and A alone in 15 days, then in how many days can B and C finish the work?
5. P and Q can do a piece of work in 12 days and 15 days respectively. P started the work alone and then, after 3 days Q joined him till the work was completed. How long did the work last?

Challenging Problems

6. A camp had provisions for 490 soldiers for 65 days. After 15 days, more soldiers arrived and the remaining provisions lasted for 35 days. How many soldiers joined the camp?
7. A small–scale company undertakes an agreement to make 540 motor pumps in 150 days and employs 40 men for the work. After 75 days, the company could make only 180 motor pumps. How many more men should the company employ so that the work is completed on time as per the agreement?
8. A can do a work in 45 days. He works at it for 15 days and then, B alone finishes the remaining work in 24 days. Find the time taken to complete 80% of the work, if they work together.
9. P alone can do $\frac{1}{2}$ of a work in 6 days and Q alone can do $\frac{2}{3}$ of the same work in 4 days. In how many days working together, will they finish $\frac{3}{4}$ of the work?
10. X alone can do a piece of work in 6 days and Y alone in 8 days. X and Y undertook the work for ₹4800. With the help of Z, they completed the work in 3 days. How much is Z's share?

Summary

- If two quantities are such that an increase or decrease in one quantity makes a corresponding increase or decrease (same effect) in the other quantity, then they are said to be in direct proportion or said to vary directly.
- x and y are said to vary directly if $\frac{x}{y} = k$ always, where k is a positive constant.
- If two quantities are such that an increase or decrease in one quantity makes a corresponding decrease or increase (opposite effect) in the other quantity, then they are said to be in inverse (indirect) proportion or said to vary inversely.
- x and y are said to vary inversely, if $xy = k$ always, where k is a positive constant.
- There will be problems which involve a chain of two or more variations in them. This is called as compound variation.
- By finding the proportion, we can use the fact that the product of the extremes is equal to the product of the means to find the unknown (x) in the problem.
- By using the formula $\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$, we can find the unknown (x).
- We can find the unknown (x) by Multiplicative Factor Method also.
- If two persons X and Y can do some work individually in a and b days, their one day's work is $\frac{1}{a}$ and $\frac{1}{b}$ respectively.
- X and Y together can complete the work in $\frac{ab}{a+b}$ days.